Time to Contact Estimation on Paracatadioptric Cameras

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Abstract

Time to contact or time to collision (TTC) is the time available to a robot before reaching an object. In this paper, we propose to estimate this time using a catadioptric camera embedded on the robot. Indeed, whereas a lot of works have shown the utility of this kind of cameras in robotic applications (monitoring, localisation, motion,...), a few works deal with the problem of time to contact estimation on it. Thus, in this paper, we propose a new work which allows to define and to estimate the TTC on catadioptric camera. This method will be validated on simulated and real data.

1. Introduction

The theory of time to contact (TTC) was first introduced by Lee and Young [6]. The goal of time to contact for robot navigation is to estimate the free space before the robot and, thus, to decide if the robot has to turn or to stop when the collision is imminent. This problem has been studied by many authors with a perspective camera such as [6], [1]. If we consider that such camera moves along the optical axis Z, the time to contact is defined as :

$$\tau = -\frac{Z}{\dot{Z}} \tag{1}$$

where Z is the distance of the object which is approaching the camera, and $\dot{Z}$ its velocity.
Let us consider a perspective camera of focal length f, and let us note $m(x, y, f)$ the projection of a 3D point $M(X, Y, Z)$ on the camera plane. Thus:

$$x = f\frac{X}{Z} \quad \text{and} \quad y = f\frac{Y}{Z} \tag{2}$$

Differentiating the equation of perspective projection (2), the TTC formula on perspective image is:

$$\tau = \frac{x}{\dot{x}} = \frac{y}{\dot{y}} \tag{3}$$

where $(\dot{x}, \dot{y})$ is the motion field in the camera plane. The existing methods can be decomposed to three classes:
1)Optical flow based time-to-contact: [2].
2) Gradient-based time-to-contact: [4].
3) Time-to-contact from closed contours: using Green's theorem [7].
In this paper, we propose to estimate the TTC thanks to a catadioptric camera. A lot of works deal with the problem of TTC estimation with classical perspective camera, whereas few methods have been developed for catadioptric cameras. If we consider these sensors as spherical cameras, some works [8] can be used. In 1991, Tistarelli and Sandini [11] have described an estimation of time to contact using particular sensors based on polar and log polar representations. This method consists in using optical flow as a continuous function. Kazuaki Kondo et al. [5] employ this function using HBP mirror, but this method does not propose any adaptation of TTC in a paracatadioptric image. Thus, in order to compute TTC with a catadioptric camera, the aforementioned authors use the spherical equivalence of this type of image. We discuss in this paper the problem of estimating TTC maps, we propose to work directly in the image plane by using optical flow. We will see that due to the introduction of a mirror, perspective projection (2) is not appropriate to project 3D points to the image plan. So, equation (3) is not valid for the paracatadioptric camera. Firstly, we have to define a new formulation of TTC for omnidirectional images which takes into account the introduction of the mirror. In this formulation, we have to compute the optical flow, but for the same reasons, the classical optical flow methods are not valid. Thus, we will see how to estimate the op-

tical flow in order to take into account the distortions introduced by the mirror. Finally, we will show some results with simulated and real image sequences.

2. TTC and paracatadioptric cameras

Let P be a fixed obstacle of coordinates (X, Y, Z) in the environment space $\mathbb{R}^3$. The image point is obtained thanks to a double projection. First, a 3D point $P(X, Y, Z)$ is projected on the mirror on $m_p(x_p, y_p, z_p)$.

$$m_p : \begin{cases} x_p = \frac{h\ X}{\sqrt{X^2+Y^2+Z^2}-Z} \\ y_p = \frac{h\ Y}{\sqrt{X^2+Y^2+Z^2}-Z} \\ z_p = \frac{h\ Z}{\sqrt{X^2+Y^2+Z^2}-Z} \end{cases} \quad (4)$$

Then, as the projection between the mirror and the image plane is orthographic (see Fig.1), we obtain the image point $m(x, y)$:

$$m : \begin{cases} x = \alpha_x x_p + x_0 \\ y = \alpha_y y_p + y_0 \end{cases} \quad (5)$$

Where (x_0, y_0) are coordinates of a principal point and α_x (respectively α_y) is a scale conversion mm to pixel in X direction (respectively Y direction).

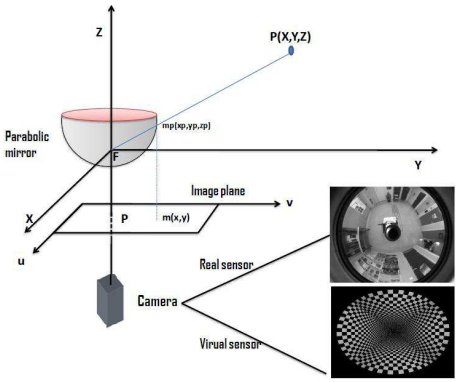


Figure 1. Paracatadioptric camera

Nevertheless, due to this double projection, (3) becomes invalid. So, we need to find a new equation which connects the TTC with $m(x, y)$. To this aim, let us consider a robot which is moving in a straight direction. Without loss of generality, we can suppose that this direction corresponds to X axis. If not, we can estimate the direction using [10] and rotate the camera axis relatively to this direction. Thus, the TTC equation is defined by:

$$\tau = -\frac{X}{\dot{X}} \quad (6)$$

Thanks to (4), we can write:

$$\frac{x_p}{y_p} = \frac{X}{Y} \quad (7)$$

Then, by derivation, we obtain:

$$\begin{aligned} \dot{x}_p &= \frac{Y(y_p\dot{X}) - (y_p X)\dot{Y}}{Y^2} \\ &= \frac{X}{Y}(\dot{y}_p + \frac{y_p}{X}\dot{X} - \frac{y_p}{Y}\dot{Y}) \end{aligned} \quad (8)$$

Such as the robot is moving along X direction, we have $\dot{Y} = 0$ and in substituting $\frac{x_p}{y_p}$ by $\frac{X}{Y}$, (8) becomes:

$$\dot{x}_p = \frac{x_p}{y_p}(\dot{y}_p - y_p\tau^{-1}) \quad (9)$$

Thus, in mirror frame, the TTC is defined by the following equation:

$$\tau^{-1} = \frac{\dot{y}_p}{y_p} - \frac{\dot{x_p}}{x_p}. \quad (10)$$

Finally by using (10) and (5), we obtain the TTC expression in the image plane:

$$\tau^{-1} = \frac{\dot{y}}{y - y_0} - \frac{\dot{x}}{x - x_0}. \quad (11)$$

Equation (11) is an adaptation of the TTC equation in a paracatadioptric sensor. Let us note that the formulation is independent of h, α_x and α_y. Equation (11) gives an expression for calculating TTC in a paracatadioptric image at each pixel. Consequently, to compute the TTC on a point $m(x, y)$, we have to estimate the apparent motion $(\dot{x}, \dot{y})$ of this current point. Once again, while a lot of methods exist in a perspective camera for optical flow estimation, due to the introduction of a mirror, these methods must be revisited. Thus, we have chosen to use an adapted framework.

3 Optical flow

Many optical flow methods have been developed in the literature. The Lucas-Kanade's method is still one of the most popular solutions for motion estimation. Radgui *et al.* [9] have shown that this method is not valid when we deal with omnidirectional image. Indeed, due to the distortion introduced by the mirror, the motion field cannot be considered constant around a neighborhood. This is why, they have proposed to adapt the Lucas-Kanade's method by replacing the constant model by the following form:

$$\begin{cases} u = a(x - x_0)^2 + a(y - y_0)^2 + c \\ v = b(x - x_0)^2 + b(y - y_0)^2 + d \end{cases} \quad (12)$$

where (x_0, y_0) is the image center and (a, b, c, d) are constants. Moreover, this neighborhood cannot be fixed

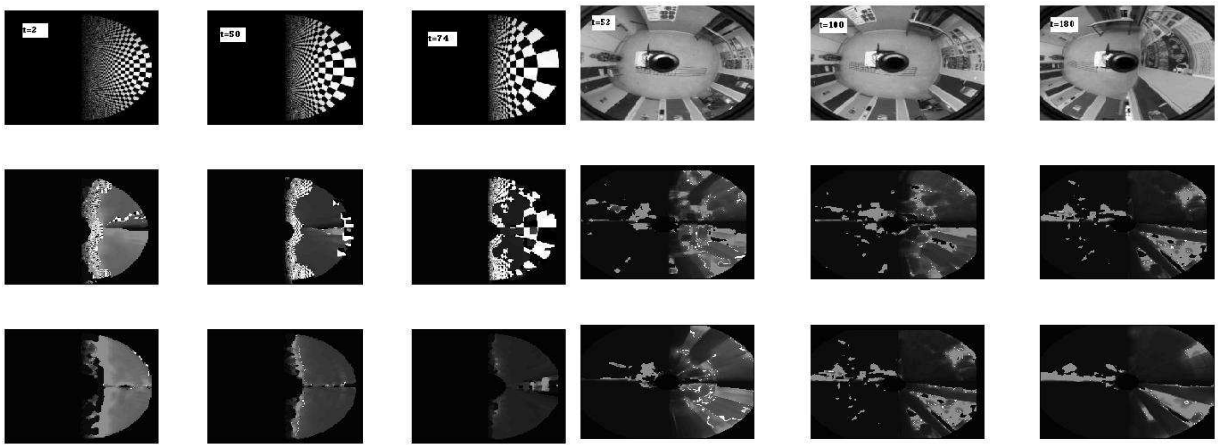


Figure 2. Estimation of TTC on synthetic and real sequence:The top row shows a progression of synthetic and real images at time. The second row shows the results of TLK and the third row shows results of TALK. Dark shading indicates short TTC, light shading shows long TTC, white shading representing the outliers value of TTC and black shading shows $TTC = 0$.

in all the image as in a perspective camera. [3] has proposed to adapt the neighborhood in order to take into account the distortion of these atypical images. In this paper, we propose a similar neighborhood.

4 Experimental results

In order to validate our method, we have tested the proposed algorithm on synthetic and real sequences.
Synthetic sequences. Our synthetic sequence is generated by Pov-ray[1] software. The sequence corresponds to a virtual robot which is moving with a uniform unidirectional motion towards X-axis. For all sequence, the intrinsic parameters [2] are fixed on $\alpha = 40$, $h = 2.3$ and $(x_0, y_0) = (100, 100)$. The main advantage is that ground truth here can be exactly known. Three of the 100 stimulus frames corresponding to the time $t = 2$, $t = 50$ and $t = 74$ are shown in the top row of Fig.**??** in columns 1, 2 and 3. So, to improve the accuracy of TTC algorithm, we are applied (11) with two different techniques based on optical flow estimation: Lucas-Kanade optical flow and adapted Lucas-Kanade optical flow. We note, that, the purpose of this paper, is not a comparison of different problem solutions of optical flow in paracatadioptric images, but rather to show how estimation of optical flow can affect accuracy of calculating of TTC. The classical neighborhood windows size used in Lucas-Kanade method is $15*15$ and equivalently the adapted neighborhood windows [3] ϑ is fixed by $d\theta = \frac{pi}{25}$ and $d\varphi = \frac{pi}{50}$. The algorithm computes TTC at every pixel. To reduce effect of outliers, we use median filter at the neighborhood of the current pixel of size $6*6$. Comparing TTC of Lucas-Kanade (TLK) with TTC adapted of Lucas-Kanade (TALK), we can see that TALK is able to estimate TTC correctly (see Fig.2), in progress in time, results shown that TTC dropped linearly, near of end of sequence ($t > 80$), the results become unreliable (see Fig. 3).

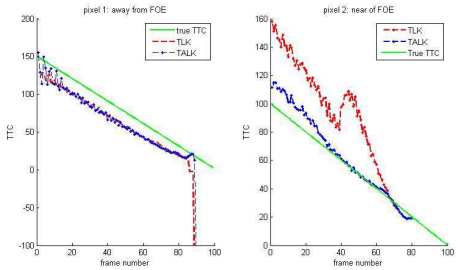


Figure 3. Left column: TTC of pixel away from FOE. Right:TTC of pixel near of FOE.

However, the main problem near the end of the sequence is temporal aliasing (i.e. missed value of TTC where objects move through each other due to discrete sampling). The behavior of the TTC expression is clearly shown in Fig.3, we applied (11) in two pixels in the image, the first pixel is chosen away from the FOE

[1] www.povray.org

[2] We need h and α only to adapt optical flow estimation. To simplify this modelisation, we consider that α_x and α_y will be equal $\alpha_x = \alpha_y = \alpha$.

(the points on the motion field where the flow vectors seem to be emerging [10] (see Fig. 4)) that the TTC is illustarted in Fig. 3, second pixel is chosen near of FOE that TTC is illustrated in Fig. 3. TLK was a bias in the estimate towards larger values than the true TTC (i.e the optical flow was overestimated) with pixel near of FOE. Thus, in the situation of translation discribed here, an other common problem to enhance estimation of TTC appears, it is the problem of expansion of optical flow. Thus, around the FOE, TTC is not defined for two reasons: first, because of the dividing on term $x - x_0$ in (11), that it becomes zero in the expansion axis. Second, in the FOE point, optical flow is bad estimated.

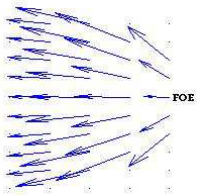


Figure 4. Example of an optical flow field with focus of expansion.

Video sequence. We have also used sequences of real omnidirectionnal images. The platform used for the experiment was a Pioneer-AT robot where we have embedded a paracatadioptric camera. The mobile robot was moving along the X-axis towards a subset of posters pinned on a wall (see Fig.2 in the top row in column 4,5 and 6). The calibration of the camera was conducted with the toolbox Hyscac[3]. Unfortunately in this case, the real TTC is not exactly known, but the speed is approximately constant ($= 12s$) and distance is equal to $3.74m$. Three of the 190 real frames corresponding to the time $t = 52$, $t = 100$ and $t = 180$ are shown in the top row of Fig.2. So, in the same way of the previous section, we applied (11) with two different techniques based on optical flow estimation: Lucas-Kanade optical flow and adapted Lucas-Kanade optical flow. Overall, the results in Fig 2 show that the TTC algorithm is more appropriate with the adapted optical flow, but severals problem can reduce the robustness of the TTC estimation, when the camera is very close to the obstacle; the main problem is due to temporal aliasing $(t > 180)$.

5. Conclusion

A new formulation of TTC using paracatadioptric images has been described in (11) and demonstrated on both synthetic and real sequences. We have presented a new method which provides dense time-to-contact maps. This method reformulates the TTC for paracadioptric in order to take into account the distortion introduced by the mirror. The results show a good behavior of our method with an adapted optical flow. As a short time perspective, we will reduce the temporal aliasing introduced by the optical flow estimation by taking into account temporal filtering of the TTC.

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